Nearly Identical Twins or Distant Cousins, Revisited. Weibull or Log-Normal Distributions to Characterize Fatigue Life Scatter – Which is Recommended?

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I was greatly honored to present the Plantema Memorial Lecture in Nagoya (2017). My lecture consisted of three topics; the middle one was entitled "*Nearly Identical Twins or Distant Cousins*?" which dealt with the differences between the Weibull and Log-Normal distributions, both of which are meant to deal with fatigue test scatter.

The current paper, which I propose to present at the 2019 ICAF Symposium, describes further developments on this topic, including a tentative conclusion on which distribution is more correct.

One of the downsides of using fatigue life methodology to determine the safe-life, is that considerable scatter exists, which needs to be accounted for in determining the life to crack initiation.

SuperSMITH Software<sup>1</sup> has been developed by Fulton Findings to analyze several statistical distributions, including obtaining their statistical parameters and plotting their results. This software is based on the theoretical methods described in the "New Weibull Handbook"<sup>2</sup>.



Figure 1. Weibull analysis performed by SuperSMITH Software<sup>1</sup> indicates a Safe-Life of 86 KC (86,000 cycles) for a typical application.



Figure 2. Log-Normal analysis performed by SuperSMITH Software<sup>1</sup> indicates a Safe-Life of 300 KC (300,000 cycles) for the identical application.

By comparing the results shown in Figure 1 to those shown in Figure 2, vast differences in the calculated safe-life exist between the two distributions. Both cannot be correct!

Due to these great differences, the author developed a method to help determine which distribution is more accurate. A large fatigue-life database was constructed using *test results* of many fatigue tests, all composed of various aluminum alloys. In total, the database consisted of 86 specimens.

The above 86 fatigue test results were combined to result in a single test result. The 86-fatigue test lives were normalized to result in a "Characteristic Life" of 50,000 cycles. The lowest failure among the 86 *virtual specimens* was at 18,975 cycles, while the highest failure was at 99,784 cycles. Weibull and Log-Normal plots (Figures 3 and 4) were built for the 86-specimen database, again using SuperSMITH Software<sup>1</sup>.



Figure 3. Weibull plot for the 86-specimen test, indicating a safe-life of about 4,700 cycles.



Figure 4. Log-Normal plot for the 86-specimen test, indicating a safe-life of about 13,000 cycles.

The two safe-life calculations differed by a factor of 2.7, which is unacceptable!

In order to analyze which is the more accurate distribution, the data of Figures 3 and 4 were replotted on log-log axes, as is shown in Figure 5.

Figure 5 indicates that six of the eight *earliest failures*, which are below the 10% probability of failure line, clearly indicated that the Weibull distribution is more accurate than the Log-Normal distribution, when

dealing with fatigue failures. In addition, two other considerations point to additional benefits of utilizing the Weibull distribution when dealing with fatigue-life failures to determine the safe-life.

(All these considerations will be explained fully in the final paper.)

In view of these results, the author concluded *(tentatively)* that the Weibull distribution should be preferred to determine the safe-life of a fatigue-critical structural member that has undergone fatigue testing.



Figure 5. Weibull and Log-Normal plots for the 86-specimen test.

References

- 1. SuperSMITH Software, release 5.0-CY, developed by Fulton Findings.
- 2. R. B. Abernethy, "The New Weibull Handbook", 5<sup>th</sup> edition, 2009.