Stress-Intensity Factor Solutions for Tapered Lugs with Oblique Pin Loads

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This paper summarizes five years of research into new stress-intensity factor (SIF) solutions for radially advancing cracks at holes in tapered and obliquely loading lugs, as illustrated in Figure 1. To our knowledge, these solutions are the first direct methods capable of analyzing these fracture critical connections. The new SIF solutions rely on weight function (WF) methods, remain tractable for engineering fatigue and fracture analyses, and span a very wide range of applicable geometries (including loading angles between 0 and 90 degrees and independent lug taper angles between 0 and 90 degrees). These solutions support cracks on either side of the pin hole (the "short ligament" or "long ligament" sides. For cracks on long ligaments, users can select the crack plane based on maximum principal stresses or Mises stresses. These solutions are verified by a large database of independent high-fidelity three-dimensional finite element analyses (FEA) generated using an automated approach that accelerates the evaluation process while ensuring solution quality. This work also discusses the underlying sensitivity studies used to determine appropriate boundary conditions and material properties.

Our efforts examine obliquely pin-loaded and tapered lugs using advanced computational methods to extract high-fidelity SIF solutions for semi-elliptical corner cracks and straight through cracks. We adopt a nonlinear contact detection scheme coupled with the surface-to-surface ("mortar") method to apply tractions at interfaces. This numerical scheme supports contact between a deformable pin in contact with a deformable lug. Our studies show that assumptions about bearing stress profiles commonly employed in legacy SIF solutions for pin-loaded holes (for example, cosine squared distributions) can be overly conservative under some conditions. Instead, our approach directly calculates and then applies the actual contact stresses in uncracked pin-lug connections. By modeling deformation, these analyses demonstrate the interaction between the pin and lug, e.g., bearing stresses generated by the same pin loading in a relatively wide or narrow lug. The numerical scheme can be extended to model complicated physical processes with little effort, as we illustrate by stress sensitivity studies of pin-lug stiffness mismatch and different coefficients of Coulomb friction. Critically, the numerical method supports all possible loading angles between vertical and horizontal. The current studies focus on neat-fit pin-lug connections without residual stresses or pin bending, but the numerical contact scheme will also support other geometric variations: interference fits, clearance fits, bending, and residual stresses due to cold expansion of the hole.

These investigations also illustrate rapid automation capabilities to build, execute, and process models with complicated geometry and loading. Here, the Python scripts directly interface with the Abaqus Computer Aided Engineering (CAE) graphical user interface (GUI) to produce pin-loaded lug models with and without cracks. These scripts enable analyses to be built using the same commands available to build models manually. Scripts adopt the same workflow used in the GUI, which enables rapid debugging and post-analysis modifications through the GUI. Furthermore, these scripts support post-processing of analyses to extract stresses, SIFs, and other quantities of interest. These scripts increase the efficiency of an analyst to develop analysis sets of hundreds to thousands of geometries; for example, 2D analyses can be built in a few seconds rather than several minutes. This greatly improved efficiency enables improved solution quality during model development: increased mesh refinement, better mesh control, and precise partitioning of the near-tip region.

Informed by these analyses, this work demonstrates several key features of stresses from uncracked tapered lugs with neat-fit pins. Stiffness mismatch has a negligible impact on stresses in pin-loaded lugs. Higher values of the Coulomb friction coefficient lead to higher stresses near the pin hole and lower stresses near the opposite free surface. In these studies, we assume that cracks form at either the maximum opening stress location or the maximum Mises stress location. Since cracks may form on the shorter ligament or the longer ligament, these SIF solutions support both cases. As shown in this work, the crack plane is rarely 90 degrees from the loading direction and may reach nearly 150 degrees for some loading angles and lug tapers. On the critical stress plane, our analyses illustrate stress variations due to the relative pin-diameter/lug-width, oblique loading angles, taper angle, and height. These analyses lead to the development of routines that predict critical stress plane angles and stress gradients for most realistic combinations of pin diameter, lug radius, lug height, taper angle, and oblique loading angle.

These stress analysis results enable new and unique SIF solutions for through cracks or corner cracks at pin-loaded holes to be derived using WF methods. WF methods compute SIFs using integral kernel functions (the weight function) for the appropriate geometry and stress gradients on the crack plane in the corresponding uncracked body. In general, it is much easier to compute these uncracked stress profiles than to compute the actual stress intensity factor using 3D FEA methods, and the resulting database of stresses can easily be interpolated to span a very wide solution space. Furthermore, WF solutions can easily accommodate residual stresses from cold expansion, thermal stresses, and other loading features that may prove intractable using dense tables of numerically calculated SIFs. WF solutions extend the range of non-dimensional geometric parameters (*e.g.*, crack ellipse shape) and can support additional loading scenarios (*e.g.*, in-plane bending).

Extensive verification of these WF solutions supports the credibility of the new SIFs. The verification process generates independently calculated FEA SIFs for through cracks and corner cracks. This verification process creates the SIF database using the same Python scripting interface described earlier to automate model development, execution, and extraction of key results. Due to the large solution space, the verification process employs a modified Latin Hypercube Sampling (LHS) to ensure a space filling design of experiment. LHS covers the entire solution space, including common geometries (e.g., near-circular crack shapes) and extreme configurations (e.g., cracks with extremely elongated elliptical shapes). LHS enables analysts to define a computational "budget" (number of analyses) and then supplement analyses in regions of uncertainty. The large number of analyses also supports statistical measures of the goodness-of-fit: predicted vs. actual plots, histograms, and rank ordering of deviations between WF and independent SIF results. This information may be useful for uncertainty quantification. These new SIFs show improved agreement with benchmark FEA SIFs compared to legacy solutions for straight lugs with straight loading.

Experimental datasets of fatigue crack growth for Al 7075-T651 and 4340 steel at two R-ratios support validation of the improved lug solutions for straight lugs. The new lug solutions predict lives within 50% of mean experimental values and have less scatter than predictions based on legacy solutions.

Sensitivity studies illustrate differences between lives computed using new and legacy SIF solutions. These differences may be large, especially when the pin hole is a significant fraction of the outer lug radius. Additional studies illustrate differences between lives computed with the new solutions if cracks form on the short ligament or long ligament. For cracks on the long ligament, these studies also illustrate life differences using the maximum principal stress location or the Mises stress location for crack formation.

The powerful new methods used here to generate SIFs for tapered lugs—automated FEA generation of tables of stresses in uncracked bodies, interpolation of these stresses in WF formulations, and automated verification using independent benchmark FEA SIF solutions—can easily be extended to build reliable new SIF solutions for other complex combinations of loads and cracked geometries.



Figure 1. Illustration of FEA for cracked tapered loaded lugs (left). The geometric idealization of the tapered lug is shown for cracks on the short ligament (center) and long ligament (right).

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